A Digital Signal-Processing Approach for Phase Noise Measurement

L. Angrisani, M. D'Apuzzo, and M. D'Arco

***Abstract*—A digital signal-processing method for phase noise measurement is presented. By properly over-sampling the input signal and adopting an optimized quadrature demodulation scheme, the method grants acceptable performance in analyzing sinusoidal carriers, the frequencies of which range from fractions of hertz up to hundreds of megahertz. Moreover, the method shows itself a valid alternative both to analog measurement systems, especially for the evaluation of close-to-the-carrier phase noise and time interval analyzers, particularly for carrier frequencies greater than a few units of megahertz.**

At first, the fundamental stages of the proposed method are described in detail. Its theoretical performance is then derived and compared to that offered by other measurement solutions already available on the market. Finally, the results of experiments carried out on actual signal sources are presented.

***Index Terms*—Over-sampling, phase noise measurement, quadrature demodulation, signal source characterization, time interval analyzers.**

I. Introduction

As it is well known, phase noise consists of undesired random fluctuations distorting the linear trend of the phase of a sinusoidal carrier provided by an actual source. It is a significant performance factor in many application fields such as space telemetry systems, Doppler radar, radio and television systems, or communication links. As a matter of fact, poor phase noise limits the precision of satellite positioning, reduces the operating range of radar, degrades the quality of television pictures, and spoils the quality of data transmission [1], [2].

Currently, several methods are available for measuring phase noise. Specifically, methods based on the use of i) dedicated systems (also called phase noise measurement systems), ii) spectrum analyzers, iii) time interval analyzers, and iv) interferometric approaches can be distinguished. They differ from one another in measurement principle, performance, and carrier frequency range [3]–[6].

Dedicated systems and spectrum analyzers are analog systems allowing the analysis of carriers, the frequencies of which typically range from 50 kHz up to about 2 GHz. This upper limit sometimes reaches 26 GHz thanks to the use of a downconverter, which, however, degrades the overall noise-floor. In particular, dedicated systems show very good performance (noise-floor never greater than -140 dBrad^2/Hz) for frequency offsets from the carrier greater than 1 Hz. Some problems arise, on the contrary, for close-to-the-carrier frequency offsets due to the nonzero extent of phase-locked-loop (PLL) bandwidth; moreover, in the presence of signal drift during the course of the measurement, the system may go out of lock and fail [3]. With regard to spectrum analyzers, poor results are provided whenever either AM noise or PM and/or FM residuals affect the analyzed signal. In addition, the noise-floor at very low frequency offsets, generally not greater than tens of hertz, is difficult to be estimated [3].

The operating range of time interval analyzers extends from fractions of hertz up to hundreds of megahertz. Their performance strongly depends on their limited and constant timing resolution, which causes measured phase to be affected by quantization noise. Moreover, the performance degrades upon carrier frequency increasing, because of the direct proportionality between the power of the aforementioned quantization noise and the carrier frequency itself. This is the reason why the use of a downconverter is recommended when high-frequency (greater than tens of megahertz) carriers have to be analyzed. In such cases, the overall performance of the measurement system is practically fixed by the noise-floor of the downconverter itself [7].

Interferometric approaches make it possible to measure only the phase noise that some devices, such as phase shifters and isolators, add when a pure sinusoidal carrier passes through them. As recently demonstrated [8], these approaches show good performance (noise-floor of about -180 dBrad^2/Hz) both at microwave and very high frequency (>100 MHz) bands. Until now, however, it seems that no step has been taken to make this approach operative also at lower frequencies; this is probably due to the difficult realization of some custom parts of the measurement circuit [8].

The authors propose a new, cost-effective method for phase noise measurement of sinusoidal carriers in the frequency range extending from fractions of hertz up to hundreds of megahertz. In particular, its main goal is to allow both a noise floor lower than that granted by dedicated systems for close-to-the-carrier frequency offsets and a quantization noise power lower than that produced by high-performance time interval analyzers in the presence of high-frequency carriers. The method requires only an appropriate data acquisition system and a suitable processing unit. Specifically, after being over-sampled, digitized, pre-processed, and decimated, the input signal passes through an optimized quadrature demodulation scheme, the output of which gives the time-domain evolution of the phase noise affecting the analyzed carrier.

In Section II, details concerning the fundamental stages of the proposed method are given; in particular, the critical role some measurement parameters play on its overall performance

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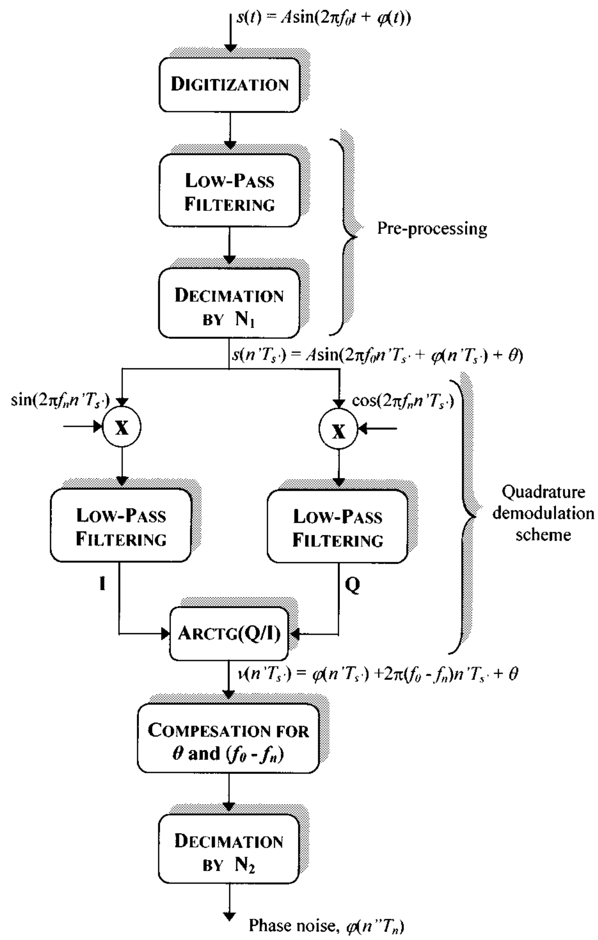

Fig. 1. Block diagram of the proposed method.

is highlighted. The third section is fully devoted to the description of the tests carried out for validating the method. In particular, the theoretical performance is first evaluated and compared to that offered by dedicated systems and high-performance time interval analyzers. Then, several experiments on signals provided by actual sources are also executed in order to assess the method's reliability and effectiveness. Finally, remarks about the obtained results are drawn in the conclusions.

II. Proposed Method

The fundamental stages of the proposed method are roughly sketched in Fig. 1. Its basic idea and operating strategies are first described in detail. Then, particular emphasis is put on some measurement choices, which greatly influence the result.

A. Fundamental Stages

At first, the sinusoidal signal (f_0, actual frequency) provided by the source under analysis is over-sampled (f_s, sample rate) and digitized by means of a data acquisition system. The samples of the acquired record are then suitably pre-processed with the aim of reducing the power of the quantization noise introduced by the digitization process; it is also possible to virtually increase the vertical resolution of the adopted data acquisition system. In particular, a low-pass filtering and a proper decimation are carried out. Both actions are performed at the same time and in a very straightforward manner. Starting from the first sample, successive sets, disjointed from one another and containing N_1 consecutive samples, are considered; for each set, the average value is calculated and retained. The resulting sample rate of the obtained signal $s(n'T_{s'})$ is equal to $f_{s'} = f_s/N_1 (T_{s'} = 1/f_{s'}); n'$ is the discrete time-variable at this point of the analysis. Moreover, a constant phase (θ in Fig. 1) has to be considered in order to account for (i) the absence of synchronization between the beginning of the digitization process and the time instants (in correspondence of which the phase of the analyzed signal equals a multiple of 2π), and (ii) the time delay introduced by the pre-processing stage.

A suitable scheme, generally adopted for coherent demodulation of phase-modulated (PM) signals, is then applied to the aforementioned $s(n'T_{s'})$. Two branches can be distinguished, the **I** branch and **Q** branch (Fig. 1). With regard to the I branch, $s(n'T_{s'})$ is multiplied by a sinusoidal carrier, the frequency of which is equal to the nominal frequency f_n of the input signal. The multiplication gives rise to both a base-band component and a high-frequency component. A low-pass filtering is carried out in order to retain only the base-band component (also called the **I** component). As for the **Q** branch, the orthogonal version (also called the **Q** component) of the aforementioned base-band component is furnished as a result of similar operations. Hence, the time-domain evolution of the phase noise $\varphi(n'T_{s'})$ is simply retrieved by, first, evaluating the sequence $v(n'T_{s'})$ (Fig. 1) according to

$$\begin{aligned} v(n'T_{s'}) &= \arctan\left(\frac{Q}{I}\right) \\ &= \varphi(n'T_{s'}) + 2\pi(f_0 - f_n)T_{s'}n' + \theta \end{aligned} \tag{1}$$

and, then, compensating for the constant phase, θ, and the frequency deviation, $f_0 - f_n$. Specifically, the compensation is achieved by subtracting from the obtained $v(n'T_{s'})$ the best-fit straight line, suitably calculated. Moreover, it is worth highlighting that an unwrapped, four-quadrant inverse tangent has to be used in (1).

Taking into account that the bandwidth of the low-pass filters adopted in the demodulation scheme always has an extent lower than f_n, the obtained sequence $\varphi(n'T_{s'})$ is further decimated in order to eliminate redundancies. In particular, the value of the decimation factor N_2 is chosen in such a way that the resulting sample rate is equal to $f_n (T_n = 1/f_n)$. With reference to Fig. 1, n'' is the discrete time-variable after this second decimation process.

B. Measurement Choices

Attention is paid mainly to (i) the parameters of the data acquisition system and (ii) the frequency response of the two low-pass filters adopted in the demodulation scheme. For the sake of clarity, all presented results are related to an application example involving the following simulated carrier

$$s(t) = A\sin(2\pi f_0 t + \varphi(t)) \tag{2}$$

with the hypothesis that A is equal to 1 V, both f_0 and f_n are equal to 1 MHz, and $\varphi(t)$ is a flicker noise with an absolute value comprised within 8 mrad and a bandwidth of about 250 kHz [9].

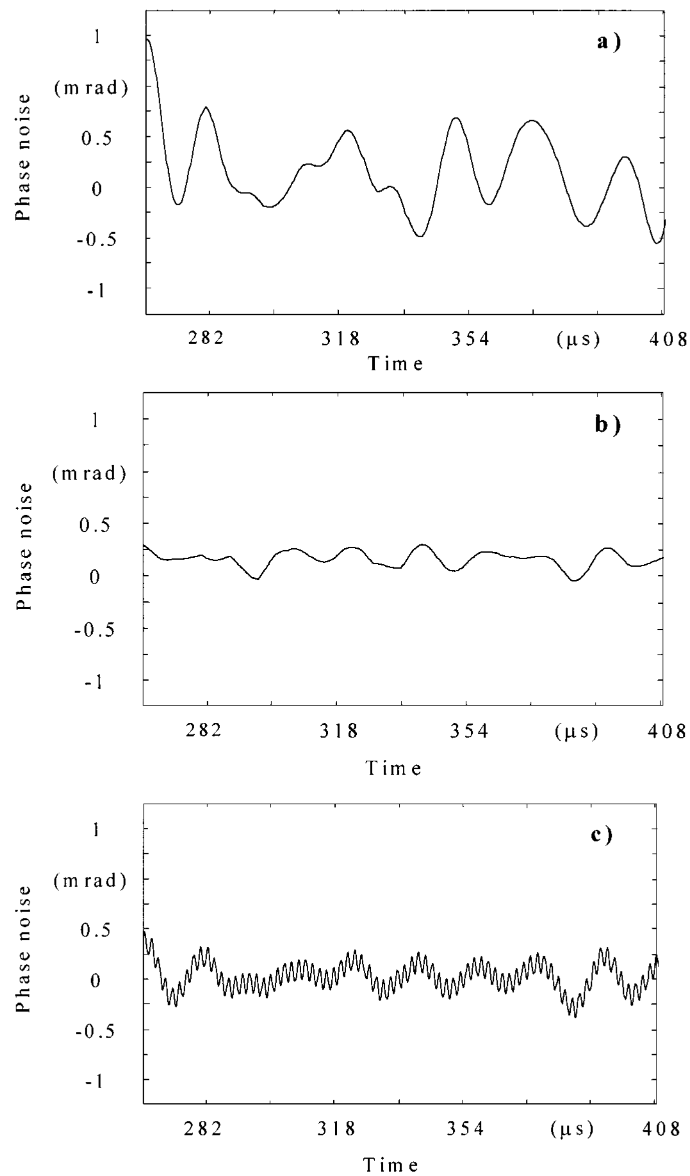

Fig. 2. Portion of the difference between measured phase noise and nominal one in the presence of a vertical resolution of the data acquisition system and a transition band of the low-pass filters in the quadrature demodulation scheme characterized respectively by (a) 8 bits and $f_n/6$ extent, (b) 11 bits and $f_n/6$ extent, (c) 11 bits and zero extent.

1) Concerning (i): Fig. 2(a) and (b) are given to highlight how the quantization noise affects the measurement results provided by the proposed method. These figures show a portion of the time-domain difference between the phase noise obtained from the application of the method to the signal in (2) and the nominal one $\varphi(t)$, regardless of the final decimation by N_2. Specifically, the results presented in Fig. 2(a) have been obtained by applying the quadrature demodulation scheme presented above to the considered signal with the hypothesis that it has been digitized at 16 Msamples/s with 8-bit vertical resolution.

The time-domain trace given in Fig. 2(b) has been obtained by applying the method to the same signal, with the hypothesis that it has been over-sampled at 2 Gsamples/s with 8-bit vertical resolution. The factor N_1 adopted in the pre-processing stage has been equal to 125 in order to obtain the same sample rate of the previous case; this choice allows the vertical resolution to be increased (Δ_{bit}) by more than 3 bits, according to the well-known relation

$$\Delta_{\text{bit}} = \frac{\log_2 N_1}{2}. \tag{3}$$

Comparing the results, it can easily be noted (as expected) that the higher the vertical resolution, the lower the difference between the measured phase noise and the nominal one, and, consequently, the better the measurement accuracy.

Taking into account the specifications of the most significant data acquisition systems currently available on the market, two alternatives can be made for limiting the effect of the quantization noise, thus improving the performance of the method. On the one hand, an excellent vertical resolution (16-bits or more) could be pursued, even though to the detriment of the maximum sample rate allowed. On the other hand, a high sample rate (up to units of gigahertz) could be preferred; the low vertical resolution (generally 8-bits) is then enhanced by proper pre-processing and decimation stages. In the first case, very good accuracy is granted in analyzing sinusoidal carriers, the frequencies of which are comprised in a limited frequency range. In the second case, this frequency range is much larger than the previous one, even though the accuracy degrades upon increasing the carrier frequency.

With the aim of pursuing as large a frequency range as possible, the authors' choice has fallen on the second alternative. The choice has also been encouraged by the reduced number of samples at each period of the analyzed carrier N_2 needed by the demodulation scheme for its proper operation. Specifically, the results of tests conducted on several carriers have confirmed that the enhancement of measurement accuracy attainable from the direct application of the demodulation scheme to the over-sampled signal is much less than that achievable with pre-processing and N_2 ranging within the values of 15 to 25.

2) Concerning (ii): With regard to the frequency response of the two low-pass filters adopted in the demodulation scheme, some remarks are necessary. First, the extreme of their transition band must be lower than f_n. This is necessary to avoid the undesired signals, oscillating at frequencies very close to f_n, and due either to a voltage offset or drift superimposed on the input carrier, that could affect the measurement results.

Moreover, Fig. 2(b) and (c) emphasize the influence of the shape of the aforementioned frequency response on measurement results. In particular, the results presented in Fig. 2(b) have been obtained by adopting a finite impulse response (FIR) filter, the frequency response of which is the best approximation of that trapezoidal response characterized both by a pass band equal to $f_n/2$ and a transition band extending as far as $2f_n/3$. The results depicted in Fig. 2(c) have been obtained by adopting another FIR filter, the frequency response of which is the best approximation of that ideal, low-pass response characterized by a cut-off frequency equal to $f_n/2$.

With the aim of establishing which frequency response can produce the best accuracy, many tests have been conducted on several carriers that are different from one another both in nominal frequency and phase noise. The results have demonstrated that the FIR filter consisting of 512 taps, and approximating, at best, the previously described trapezoidal response has to be adopted.

III. Performance Assessment

In the literature, the performance of both dedicated systems and time interval analyzers in phase noise measurement is generally given with reference to theoretical operating conditions. The typical noise floor of dedicated systems is expressed as the combination of the noise generated only by the phase detector, base band amplifier, and base band analyzer with the hypothesis that these operate in a nominal condition [3]. Concerning time interval analyzers, only the power of the quantization noise due to the nominal timing resolution is taken into account and displayed versus carrier frequency [7].

To attain a comparison on the same level, the performance of the proposed method has been assessed with the hypothesis that the data acquisition system used offers its specified performance. In particular, the results obtained refer to the Tektronix model TDS 540™ (Tek 540™) digitizing oscilloscope, characterized by the nominal sample rate of 2 Gsamples/s and a vertical resolution of 8-bits. This data acquisition system is capable of directly executing the described pre-processing stage in its "high-resolution" acquisition mode [10]; moreover, it is used for carrying out the experiments described in the next section.

To assess the performance of the proposed method, numerical tests have been conducted on several sinusoidal carriers that are different from one another both in frequency and phase noise. In particular, flicker noise has been considered, characterized by a bandwidth never larger than $f_n/4 (f_0 = f_n)$ and an rms value always ranging within a few units of milliradians. The maximum sample rate allowed by the Tek 540™ is used as the generation rate; moreover, the specified jitter affecting the sampling process is applied, and an ideal 8-bit quantizer is assumed. Taking into account the behavior of the Tek 540™ when operating in its "high-resolution" mode, the value of N_2 has always been within the range of 15 to 25.

To properly draw a comparison between the proposed method and dedicated systems, the noise floor has been evaluated. To this aim, the power spectral density of the time-domain difference between the phase noise measured by the method and the originally superimposed one, assumed as reference, has been evaluated. It has been verified that the estimated power spectral density is independent both of the frequency of the analyzed carrier and the phase noise affecting it. The results are displayed in Fig. 3 versus frequency offset from the carrier; the typical noise floor of high-performance dedicated systems [3] is also given. It is worth noting how the noise floor of the proposed method

— is roughly uniform (-150 dBrad^2/Hz) in a range of frequency offsets (10^1–10^4 Hz);
— rapidly decreases for frequency offsets greater than 10^5 Hz;
— is lower than that of dedicated systems for close-to-the-carrier frequency offsets (<0.7 Hz).

To compare the proposed method to time interval analyzers, the power (square of the rms value) of the aforementioned time-domain difference has been evaluated. As verified, the estimated

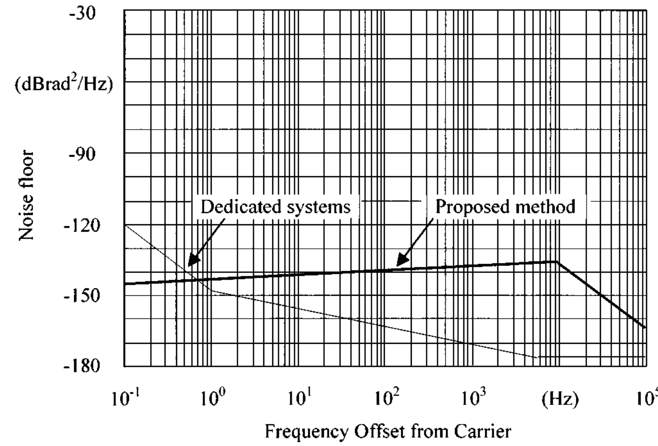

Fig. 3. Noise floor of the proposed method is compared to the typical one offered by dedicated systems.

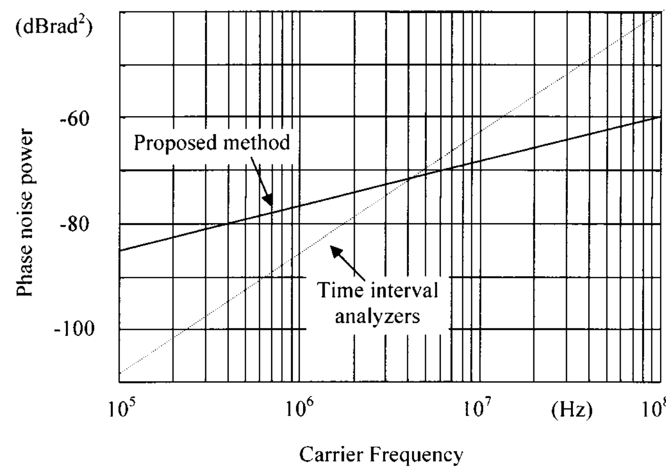

Fig. 4. Performance of the proposed method is compared to the typical one offered by time interval analyzers.

power grows linearly, on a logarithmic scale, upon increasing the carrier frequency. In Fig. 4, the results obtained are displayed versus carrier frequency along with the theoretical performance of a time interval analyzer having 50 ps timing resolution [7]. It can be observed that

— time interval analyzers have to be preferred for carrier frequencies lower than a few megahertz;
— the proposed method provides acceptable performance also in the presence of very high carrier frequencies, thus reducing the frequency range for which a down-converter is needed.

IV. Application to Actual Sources

The proposed method has also been applied to sinusoidal carriers provided by actual sources. These tests have aimed at verifying that measurement results fully agree with the specifications declared by manufacturers. For the sake of brevity, only some results related to the HP model 33 120A™ arbitrary waveform generator [11] as well as the PHILIPS model PM

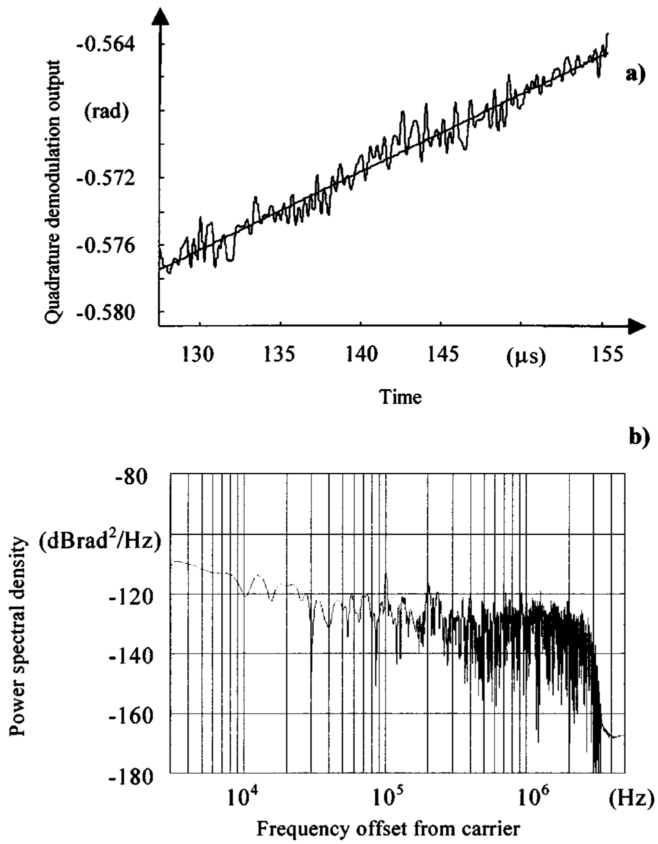

Fig. 5. (a) Measured phase noise affecting a 10 MHz sinusoidal carrier provided by a HP model 33 120A™ arbitrary waveform generator; (b) its power spectral density.

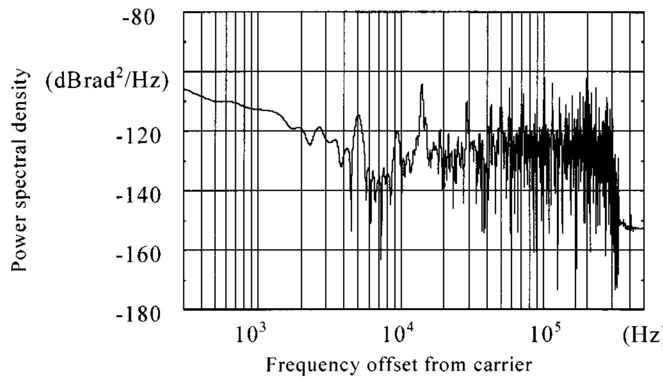

Fig. 6. Power spectral density of the phase noise affecting a 1 MHz sinusoidal carrier provided by a Philips model PM 5193™ function generator.

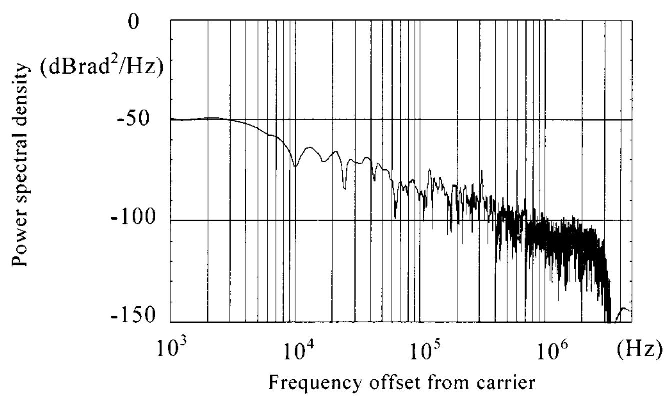

Fig. 7. Power spectral density of the phase noise affecting a 10 MHz sinusoidal carrier provided by a Philips model PM 5193™ function generator.

5193™ function generator (programmable synthesizer) [12] are described.

According to the specifications of the first source, the produced phase noise is lower than -55 dBc in a 30 kHz frequency band. In equivalent terms, phase noise lower than -97 dBrad^2/Hz is expected. As for the second source, its specifications declare that phase noise lower than -80 dBc/Hz (-77 dBrad^2/Hz) is generated if the frequency of the sinusoidal carrier is lower than 2 MHz.

Fig. 5 refers to a 10 MHz sinusoidal carrier provided by the HP model 33 120A™ arbitrary waveform generator. In particular, Fig. 5(a) gives a portion of the time-domain trace furnished by the proposed method; its roughly linear trend is due to a constant deviation of the frequency of the analyzed carrier from its nominal value. A best-fit straight line is then evaluated and subtracted from the aforementioned trace. The remaining trace gives the wanted phase noise; the slope of the straight line can be useful for estimating the frequency deviation (73 Hz). Figure 5(b) presents the power spectral density of the phase noise; the results obtained agree with the source specifications.

Two significant cases are finally described; they both refer to the PHILIPS model PM 5193™ function generator. In particular, Figs. 6 and 7 show the power spectral density of the measured phase noise affecting a sinusoidal carrier characterized by a frequency of 1 MHz and 10 MHz, respectively. As stated by the specifications, the previously described limits are satisfied at 1 MHz and not satisfied at 10 MHz.

V. Conclusion

A new, cost-effective method for measuring phase noise affecting sinusoidal carriers has been presented. After being oversampled, the input signal is suitably processed for reducing the influence of the quantization noise. Then, an optimized quadrature demodulation scheme is applied, and a discrete sequence representing the time-domain evolution of the phase noise is obtained. The main advantages of the proposed method can be summarized as follows:

- the obtained discrete sequence can be used for evaluating any other function or parameter of interest of the measured phase noise;
- measurements can be made in the presence of both modulation and drift, thus overcoming a notable limit of dedicated systems;
- no reference signal is needed.

Experiments conducted on the sinusoidal carriers provided by actual sources have assessed the reliability and efficiency of the method.

The on-going activity is mainly oriented to enhance the performance of the method by designing a more appropriate low-pass filtering for both the pre-processing stage and the demodulation scheme. Furthermore, time-frequency as well as time-scale approaches for a more suitable analysis of the measured phase noise are also going to be applied [13].

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